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The forth sentence of §7 and comment 28 at the end of the reference list are rather misleading. I here describe them more clearly and accurately.

1. The forth sentence of §7 on page 1165 "J and $\Delta\mu$ are renormalized[28] by e-e interactions" should read "J and $\Delta\mu$ are renormalized[28] by interactions." Namely, the term "e-e" should be removed because it makes the sentence misleading.

2. To describe the exact meaning of the above sentence, comment 28 at the end of the reference list on page 1165 should read "By renormalization we here mean the following things. In the TL liquid case, we mean that the value of J or $\Delta\mu$ for a given value of $N_{\pm}$ deviates from that of free electrons. In the Fermi liquid case, on the other hand, we mean that the value of J or $\Delta\mu$ for a given value of q deviates from that of "free quasi particles," which are hypothetical *free* particles that have the quasi-particle mass and are characterized by eq. (2.2) with $f(k, k') = 0$. In contrast, if we look at J *for a given* $\Delta\mu$ then J is *not* renormalized in any sense. Therefore, the term "renormalization" should be used carefully."

Note that in the last sentence of §2 on page 1163 "because J and μ are *both* renormalized" the term "renomarlized" is used also in the sense of (the Fermi liquid case of) comment 28.

Clearly, these changes and notice do not affect the conclusions of the paper in any way.

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The conductance of one-dimensional interacting electron systems is calculated in a manner similar to Landauer's argument for non-interacting systems. Unlike in previous studies in which the Kubo formula was used, the conductance is directly evaluated as the ratio of current J to the chemical potential difference $\Delta\mu$ between right-going and left-going particles. It is shown that both J and $\Delta\mu$ are renormalized by electron-electron (e-e) interactions, but their ratio, the conductance, is not renormalized at all if the e-e interactions are the only scattering mechanism. It is also shown that nonequilibrium current fluctuation at low frequency is absent in such a case. These conclusions are drawn for both Fermi liquids (in which quasi-particles are accompanied with the backflow) and Tomonaga-Luttinger liquids.

I. INTRODUCTION

The two-terminal conductance G of one-dimensional (1D) electron systems without any scattering is given by the Landauer formula [1]

$$G = s/2\pi \tag{1}$$

where s denotes the degeneracy (which is usually 2 due to the spin degeneracy), and we take $e^2 = \hbar = 1$ throughout this paper. When electron-electron (e-e) interactions are introduced, the electron system behaves as a so-called Tomonaga-Luttinger (TL) liquid. [2–10] Early theoretical studies [7–10] indicated that the conductance of a TL liquid would be

$$G = \gamma s/2\pi, \tag{2}$$

where γ is a constant which depends on the e-e interactions. [7–10] However, the result of a recent experiment [11] agrees with eq. (1) rather than eq. (2).

To explain this discrepancy, several theoretical studies based on the Kubo formula [12] were reported. [13–15] However, they depend on specific models [13–15] and/or specific approximations. [15] Moreover, although Kawabata [15] stressed that one must carefully distinguish between the external and internal fields, this point was not clear in most work. [7–10,13,14]

In this paper, we show that the formula (1) holds quite generally for 1D systems at low temperatures if the e-e interactions are the only scattering mechanism. This is explicitly shown for *both* Fermi liquids and TL liquids. The result is independent of details of models, such as values of the Landau parameters or strength of e-e interactions. Instead of using the Kubo formula, we calculate G directly by dividing the current by the chemical potential difference. This eliminates the above-mentioned difficulty caused by the difference between the external and internal fields.

We also show that the nonequilibrium noise vanishes when G is given by eq. (1), irrespective of e-e interactions.

Throughout this paper, it is assumed that the structure of the 1D system is smooth enough and the temperature is low enough, so that scattering by impurities, defects or phonons is negligible. That is, we treat "clean" 1D systems in which e-e interactions are the only scattering mechanism. We consider the case $s = 1$ because the generalization to cases of $s \geq 2$ is trivial. We assume zero temperature for simplicity.

II. NOMINAL CONDUCTANCE OF 1D FERMI LIQUIDS

It is generally believed that a 1D interacting electron system cannot be a Fermi liquid. However, it is very instructive to evaluate the Landauer conductance for a 1D Fermi liquid because (i) it provides considerable insight into the

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underlying physics, and (ii) real systems have finite length and finite intersubband excitation energies (from the occupied subband, which is considered as the 1D system, to an empty subband) and thus some real systems might be well described as a Fermi liquid. Note that the validity of eq. (1) for Fermi liquids is *never trivial* because we need to consider the backflow [16] and we cannot simply repeat Landauer's discussion [1] with the free electron mass replaced by the renormalized mass.

A Fermi liquid is characterized by the quasi-particle distribution $n(k)$ and the Fermi wavenumber k_F. [16] At equilibrium, $n(k) = n^0(k) \equiv \Theta(|k| \leq k_F)$, where Θ is a unit step function which is unity if the argument is true and zero otherwise. Under a small external perturbation, n would change slightly as

$$n(k) = n^0(k) + \delta n(k), \tag{3}$$

and we may expand the corresponding change of the total energy E as

$$\delta E = \sum_k \epsilon_k^0 \delta n(k) + \frac{1}{2L} \sum_k \sum_{k'} f(k, k') \delta n(k) \delta n(k'), \tag{4}$$

where L is the normalization length (system size). [17] By definition, we can take f to be symmetric;

$$f(k, k') = f(k', k). \tag{5}$$

The quasi-particle energy ϵ_k is the functional derivative of E:

$$\epsilon_k\{n\} = \epsilon_k^0 + \frac{1}{L} \sum_{k'} f(k, k') \delta n(k'), \tag{6}$$

where by $\{n\}$ we indicate that ϵ_k is a functional of $n(k)$'s.

Let us evaluate the current J when $n(k)$ is of the form of the "shifted Fermi distribution";

$$n(k) = \Theta(|k - q| \leq k_F), \tag{7}$$

where q is a small ($|q| \ll k_F$) wavenumber. We will argue in the next section that this shifted-Fermi state corresponds to the experimental situation under the assumptions that the sample is well fabricated and the temperature is low enough.

We note that the above distribution is the same as what we would find if we observed the equilibrium distribution $n^0(k)$ from a moving frame. The current for such a case was calculated in §1.2c of ref. [16]. The current carried by a quasi-particle of momentum k is given by

$$j_k = \frac{1}{L}\left[v_k - \frac{1}{L} \sum_{k'} f(k, k') \frac{\delta n(k')}{\delta q} \right] \tag{8}$$

$$\simeq \frac{v_k}{L} + \frac{1}{2\pi L}[f(k, k_F) - f(k, -k_F)], \tag{9}$$

where $v_k = \partial \epsilon_k^0 / \partial k$ is the velocity of the quasi-particle, and we have used, in the second line, the fact that δn is nonzero only near the Fermi points $k = \pm k_F$. The total current is thus given by

$$J = \sum_k \delta n(k) j_k \tag{10}$$

$$\simeq \frac{q}{\pi}[v_F + \frac{1}{2\pi}(f_{++} - f_{+-})], \tag{11}$$

where $f_{++} \equiv f(k_F, k_F) = f(-k_F, -k_F)$ and $f_{+-} \equiv f(k_F, -k_F) = f(-k_F, k_F)$. For a parabolic ϵ_k^0, for example, we have $v_k = k/m^*$ and $v_F = k_F/m^*$, with m^* being the renormalized mass, and eqs. (9) and (11) show that the current is the sum of this trivial contribution and the backflow term. At first sight, the backflow may appear to modify the Landauer formula. However, this is not the case, as we will now show.

Suppose that we add a right-going ($k > 0$) quasi-particle of minimum allowable energy. The energy cost defines the chemical potential μ_+ of right-going quasi-particles. We can also define μ_- for left-going ($k < 0$) quasi-particles. From eqs. (3), (6) and (7) we have

$$\mu_\pm = \epsilon_{\pm k_F + q}\{n^0 + \delta n\} \tag{12}$$

$$\simeq \epsilon^0_{\pm k_F + q} + \frac{1}{L}\sum_{k'} f(\pm k_F, k')\delta n(k') \tag{13}$$

$$\simeq \epsilon^0_{k_F} \pm v_F q + \frac{q}{2\pi}(-f_{\pm-} + f_{\pm+}). \tag{14}$$

Hence the chemical potential difference

$$\Delta\mu \equiv \mu_+ - \mu_- \tag{15}$$

$$\simeq 2q[v_F + \frac{1}{2\pi}(f_{++} - f_{+-})]. \tag{16}$$

By dividing J by $\Delta\mu$, we obtain a conductance G, which we here call the "nominal conductance" because the above discussions did not consider structures of real samples. We will show in the next section that G equals the observed conductance G_{obs}.

For the nominal conductance, eqs. (11) and (16) yield

$$G \equiv J/\Delta\mu = 1/2\pi. \tag{17}$$

We thus find that G is *not* renormalized by e-e interactions because J and μ are *both* renormalized *by the same amount.*

III. 1D FERMI LIQUID CONNECTED TO RESERVOIRS

Our next task is to show that $G_{obs} = G$. In the above discussions we did not consider the ends of the 1D system. Both ends of a real sample, however, are connected to two- or three-dimensional systems, which are called "contacts" or "reservoirs." [1] The left (L) and right (R) reservoirs have different chemical potentials, μ_L and μ_R, respectively. The reservoirs are large enough, so that they remain at equilibrium even in the presence of a finite current J between the reservoirs through the 1D system.

To avoid undesirable reflections at the boundaries between the 1D system and the reservoirs, real samples are usually fabricated in such a way that the boundary regions are smooth and long ($>$ Fermi wavelength). We assume that this is the case. Then, the wavefunction of a quasi-particle propagating in the 1D system will diffuse adiabatically into a reservoir, without suffering any reflection. By considering the time-reversed state of this process, we can also deduce that there exists a set of excitation modes in reservoirs, each of which evolves into a quasi-particle state in the 1D system. Through these modes, reservoir L (R) provides the 1D system with right-going (left-going) quasi-particles, of energy less than μ_L (μ_R). Therefore, the reservoirs tend to drive the 1D system to the state with the shifted Fermi distribution with $\mu_+ = \mu_L$ and $\mu_- = \mu_R$. This shifted Fermi state will be reached if it is an approximate eigenstate of the 1D system. This is indeed the case if the 1D system is of infinite length because the shifted Fermi state is the ground state in a moving frame, and hence is an eigenstate in the rest frame. (The infinite 1D system has translational invariance because we have assumed the absence of scattering by impurities, defects or phonons.) For a finite-length system such a state should also be an approximate eigenstate if the length is long enough. (In §V we will present another line of reasoning, where we will show that other states cannot be realized as a steady state.) We thus find that $G_{obs} \equiv J/(\mu_L - \mu_R) = J/(\mu_+ - \mu_-) = G$. Therefore the observed conductance is *not* renormalized either: $G_{obs} = 1/2\pi$.

IV. NOMINAL CONDUCTANCE OF TL LIQUIDS

We now evaluate the nominal conductance of clean TL liquids. Since we are interested in the conductance in the limit of low frequency and long wavelength, only low-energy excitations are relevant. The low-energy dynamics of TL liquids is described by a fixed-point Hamiltonian, which is diagonalized as [2–6]

$$H = \sum_p \omega_p \hat{b}_p^\dagger \hat{b}_p + \frac{\pi}{2L}[v_N(\hat{N}_+ + \hat{N}_-)^2 + v_J(\hat{N}_+ - \hat{N}_-)^2], \tag{18}$$

where $\hat{b}_p^\dagger$ ($\hat{b}_p$) is a boson creation (annihilation) operator, and $\hat{N}_+$ and $\hat{N}_-$ are the number operators which correspond to the numbers of right-going and left-going particles, respectively. Note that the e-e interactions renormalize the two

velocities v_N and v_J differently: $v_N \neq v_J$ for interacting systems, while $v_N = v_J$ for non-interacting systems. [2–6] Let N_p and $N_\pm$ be the eigenvalues of $\hat{b}_p^\dagger \hat{b}_p$ and $\hat{N}_\pm$, respectively. Low-energy eigenstates of a TL liquid are completely labeled by these quantum numbers, [2–6] and the eigenenergies are given by

$$E = \sum_p \omega_p N_p + \frac{\pi}{2L}[v_N(N_+ + N_-)^2 + v_J(N_+ - N_-)^2]. \tag{19}$$

The dc current averaged over the sample length L, which corresponds to the current component at low frequency and long wavelength, is given by [2–6]

$$J = v_J(N_+ - N_-)/L. \tag{20}$$

Let us consider states with $N_p = 0$ for all p and $N_+ \neq N_-$. We will argue in the next section that such a state (with $N_+ + N_- = 0$) is realized in real samples. Let us define $\mu_\pm$ as in §II. We then have

$$\mu_\pm = \frac{\partial E}{\partial N_\pm} = \frac{\pi}{L}[v_N(N_+ + N_-) \pm v_J(N_+ - N_-)], \tag{21}$$

which shows that $\mu_\pm$ depends on *both* v_N and v_J, and is renormalized by e-e interactions. However, the v_N dependence disappears if we take the difference;

$$\Delta\mu \equiv \mu_+ - \mu_- \tag{22}$$
$$= 2\pi v_J(N_+ - N_-)/L, \tag{23}$$

which is renormalized *only through v_J, just as J is.* Consequently, the nominal conductance is *not* renormalized at all by e-e interactions:

$$G \equiv J/\Delta\mu = 1/2\pi. \tag{24}$$

V. TL LIQUID CONNECTED TO RESERVOIRS

Let us explore the relationship between the nominal conductance G and the observed conductance G_{obs}. As in §III, we assume that both ends of the 1D system are connected to reservoirs of chemical potentials μ_L and μ_R, and assume that the boundary regions are smooth and long. Note that the argument in §III cannot be applied directly to TL liquids because of the lack of single-particle excitations. [2–6] Instead of modifying the argument, we take a different approach. We argue that in the linear response regime the steady state must be the state with the minimum energy among states which satisfy given external conditions. Otherwise, the system would be unstable and would evolve into a state with lower energy. For our purpose, it is convenient to take the value of J as the given external condition. (This is just a Legendre transformation of a theory in which $\Delta\mu$ is given.) It is then clear from eqs. (19) and (20) that the state with $N_p = 0$ and $N_+ + N_- = 0$ has the minimum energy for a given J. Hence such a state must be realized as a steady state. We further argue that if $\mu_{L,R}$ were not equal to $\mu_{+,-}$ then the system would be unstable because an extra flow could be induced between the 1D system and the reservoirs. Therefore, what is realized as the steady state is that with $N_p = 0$ for all p, $N_+ + N_- = 0$ and $\mu_{L,R} = \mu_{+,-}$. We thus find that $G_{obs} \equiv J/(\mu_L - \mu_R)$ $= J/(\mu_+ - \mu_-) = G$. Therefore the observed conductance is *not* renormalized either, $G_{obs} = 1/2\pi$, in agreement with experiment. [11]

VI. NONEQUILIBRIUM CURRENT FLUCTUATION

So far, we have considered the linear conductance. The well-known fluctuation-dissipation theorem [12] states that the linear conductance is related to the current fluctuation $\langle \delta J^2 \rangle$ *evaluated at equilibrium, i.e., at $J = 0$.* Its zero-frequency component vanishes at zero temperature:

$$\langle \delta J^2 \rangle_{J=0}^{\omega=0} = 0. \tag{25}$$

This result is known to generally hold because it simply states that the power spectrum of the zero-point fluctuations vanishes at $\omega = 0$ (because the energy of a zero-point fluctuation is proportional to ω). In the presence of nonzero J,

on the other hand, finite $\langle \delta J^2 \rangle_{J \neq 0}^{\omega=0}$ is generally observed even at low temperatures. [18–21] This excess noise is called the nonequilibrium noise (NEN).

The NEN can vanish in some cases, such as the case of free 1D electrons without any scattering [18,19] or the case of electrons with strong dissipation. [22–24] Interestingly, we can show that the NEN vanishes also for the 1D interacting systems discussed in the previous sections;

$$\langle \delta J^2 \rangle_{J \neq 0}^{\omega=0} = 0. \tag{26}$$

To show this, we note that $\langle \delta J^2 \rangle$ is invariant under Galilean transformations. Hence, we can calculate it in a moving frame. For each of the steady states discussed in §II and §IV, we can find a moving frame in which the state becomes the ground state. For the ground state, $J = 0$ and eq. (25) yields $\langle \delta J^2 \rangle = 0$. Therefore, it should also be zero in the rest frame. [25] The necessary conditions for this conclusion are basically the same as those of the previous sections. However, one must be more careful in avoiding a hot-electron effect because the NEN is more sensitive to the electron temperature than G is. [22–24]

Kane and Fisher [26] and Chamon et al. [27] studied the NEN of TL liquids *with barriers*. Since their discussions relied on a perturbation expansion which is good only when G is small, their results cannot apply to clean TL liquids without barriers. On the other hand, our result is non-perturbative and thus is valid for clean liquids (but is not applicable to dirty liquids).

The present new prediction, that the NEN is absent in clean TL and Fermi liquids, may be tested in careful experiments.

VII. SUMMARY AND DISCUSSIONS

We have shown that the two-terminal conductance G of 1D interacting electron systems is given by eq. (1) if the systems are clean enough. We have studied both Fermi liquids and TL liquids. It is found that the physical mechanism leading to eq. (1) is basically the same in both cases: J and $\Delta\mu$ are renormalized [28] by e-e interactions, but the renormalization factors for J and $\Delta\mu$ are the same, hence their ratio ($= G$) is *not* renormalized at all. We have also shown that the nonequilibrium noise is absent in such a case.

The necessary conditions for these conclusions are the following. (i) The 1D system is clean enough and the temperature is low enough (zero temperature has been assumed for simplicity), so that scattering by impurities, defects or phonons is negligible and e-e interactions are the only scattering mechanism. (ii) The low-energy excitation spectrum of the 1D system is gapless. (Although we have not stated this point explicitly, it is clear that our discussions rely on the absence of a gap.) (iii) The boundaries between the 1D system and the reservoirs are smooth and long ($>$ Fermi wavelength), so that undesirable reflections at the boundaries are absent. (iv) The reservoirs are large enough, so that they remain at equilibrium even in the presence of a finite current between the reservoirs through the 1D system.

When some of these conditions are not satisfied the observed conductance may deviate from eq. (1). We point out one example. If dissipation (by, say, phonon emission) is nonnegligible, the 1D system will lose any correlations over a distance $L_{\rm rlx}$, where $L_{\rm rlx}$ is the "maximal energy relaxation length", [22–24] which is generally longer than the simple dephasing length (over which an energy correlation may be able to survive). [22–24] In such a case the 1D system of length L ($> L_{\rm rlx}$) will behave as a series of independent conductors of length $L_{\rm rlx}$. One will then observe Ohm's law:

$$G_{obs} \simeq (L_{\rm rlx}/L) \times (s/2\pi). \tag{27}$$

Note added in proof: We want to stress again that our results are quite general. In fact, the arguments in §II and §III for Fermi liquids have not assumed any specific values of f's, and the arguments in §IV and §V for TL liquids have merely assumed the forms of H and J, eqs. (18) and (20), *only at the low-energy fixed point.* [4–6] Note that an *interacting* Fermi liquid, in which quasi-particles are accompanied with the backflow, might be realized in certain samples. Its conductance has been exactly calculated for the first time in this paper. In contrast, previous studies [7–10,13,14] analyzed Fermi liquids only when interactions and the backflow are absent. It is also worth noting that the result (2) of early theoretical studies [7–10] can be reproduced if we forget the renormalization of $\Delta\mu$. This is consistent with Kawabata's conclusion [15] that the neglect of renormalization of the electric field leads to eq. (2).

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